

است. ظاهراً، این رساله قسمتی از کتاب جبر و مقابله طوسی است (قربانی، ص ۲۸۰)؛

۴. رساله فی البرهان علی الضرب و القسمة، نسخه خطی این رساله در دانشگاه تهران نگه‌داری می‌شود (همان‌جا)؛

۵. رساله فی الاسطرلاب الخطی، شرف‌الدین طوسی مخترع نوعی اسطرلاب مشهور به «اسطرلاب خطی» یا «عصای طوسی» است و آن قطعه چوبی مدرج به شکل عصا بود و به همین علت آن را «عصای طوسی» می‌نامیدند. این اسطرلاب از یک ریسمان دولا و یک خط کش سوراخ‌دار تشکیل شده است. او روش ساختن و به کار بردن آن را در چند رساله بیان کرده است. با اینکه ساختن این وسیله ارزان تمام می‌شد، چون کار چندانی از آن بر نمی‌آمد، کاربرد گسترده‌ای نیافت و تا کنون نمونه ساخته شده‌ای از آن به دست نیامده است (همان، ص ۲۸۰-۲۸۱؛ دایرةالمعارف فارسی، همان‌جا).

منابع:

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سارتن، جورج، تاریخ علم، ترجمه غلامحسین صدری‌افشار، تهران، ج ۲، ۱۳۸۳ ش.

شرف‌الدین مظفر بن محمد بن مظفر طوسی ریاضی‌دان و منجم بنام قرن ششم و هفتم هجری قمری است.

در منابع، گاهی او را شرف‌الدین مسعودی نیز نامیده‌اند. از تاریخ تولد و نحوه زندگی وی اطلاعی در دست نیست، اما چنان‌که از نسبش پیداست، از اهالی طوس بوده است. شرف‌الدین، در حدود ۵۴۴ق، در دمشق تدریس می‌کرد و نزدیک به سه سال به آموزش طب پرداخت. ظاهراً، در سال‌های قبل از ۵۵۴ق، در موصل می‌زیست و در آنجا نیز به تدریس مشغول بود. از مهم‌ترین شاگردان وی در موصل کمال‌الدین بن یونس، ریاضی‌دان و منجم مشهور، بود (قربانی، ص ۲۷۷؛ خلاصه زندگینامه علمی دانشمندان، ذیل مدخل؛ دایرةالمعارف فارسی، ذیل مدخل).

شرف‌الدین اواخر عمرش را در همدان گذراند. تاریخ فوت او در منابع متفاوت است و برای آن سال‌های ۶۰۹، ۶۱۰ و ۶۱۱ق را ذکر کرده‌اند. آثار به‌جا مانده از او عبارت‌اند از:

۱. فی الجبر و المقابله، او در این کتاب درباره انواع معادلات درجه سوم تحقیق کرده و آن دسته از این معادلات را یافته که ریشه داشته‌اند و شرایط خاص ریشه‌های سایر معادلات را معین کرده است. متن این کتاب از بین رفته، اما شخصی ناشناس خلاصه‌ای از آن را فراهم آورده که نسخه خطی آن در کتابخانه ایندیانا آفیس نگه‌داری می‌شود، اما جدول‌ها و برخی شکل‌ها از آن حذف شده است (قربانی، ص ۲۷۹؛ خلاصه زندگینامه علمی دانشمندان، همان‌جا)؛

۲. جواب عن مسئله سألہ امیرالمدرسة النظامیه، موضوع این رساله تقسیم سطح یک مربع معلوم به یک مستطیل و سه ذوزنقه، به نسبت معین، است. از این رساله دو نسخه خطی در کلمبیا و نسخه‌ای دیگر در لیدن نگه‌داری می‌شود؛

۳. رساله فی الخطین الذین یقربان و لایلتقیان، نسخه خطی این رساله در استانبول نگه‌داری می‌شود و موضوع آن خط مجانب هذلولی متساوی‌القطرین

حمد رضا شمس اردکانی ve dgr.; تقویم تاریخ فرهنگ و تمدن اسلام و

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07 HAZ 2009

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xxx, 507p.; 30 cm.
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OF

SCHOLARS

IN MEDIEVAL ISLAM

150 - 1000 A.H
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* الآراء الواردة في هذا الكتاب لا تعبر بالضرورة عن رأي الناشر *

of the development of Muslim philosophy. The story itself includes a sketch of a natural classification of the sciences, a discussion of spontaneous generation, and miscellaneous scientific information. It was translated into Hebrew, and Moses ibn Joshua of Narbonne (second half of the fourteenth century) wrote a commentary upon it in 1349.

It should be noted that the idea of this romance was not entirely new. Ibn Sinā had written an allegoric *Risāla* bearing the same title. But the comparison ends here. Ibn Ṭufail borrowed the name of his heroes from Ibn Sinā, nothing more. It is said that a similar story was translated from the Greek by Ḥunain ibn Ishāq. One can conceive its genesis in Hellenistic neo-Platonic circles, yet the endless discussions of Muslim theologians could not but increase its significance. The romantic frame of the story is certainly old. It is found in an ancient Arabic tale of the Alexander cycle. But Ibn Ṭufail was the first to explore it with sufficient completeness and to develop the philosophical side of it, which after all, was the essential thing. He was the real creator of it, and it was him who drew the world's attention to it.

(2) two medical treatises (lost ?). Ibn Ṭufail gave advice to Ibn Rushd with regard to the latter's Aristotelian commentaries and to his *Kulḥyāt*.

(3) a commentary on Aristotle's *Meteorologica*.

Ibn Ṭufail was the one who suggested to al-Biṭrūjī the latter's modifications of the theory of homocentric spheres.

ṬŪSĪ (Muḥammad ibn Maḥmūd al-ṬŪsĪ)

Muḥammad ibn Maḥmūd ibn Aḥmad al-ṬŪsĪ al-Salmānī
12th c. A.D

Cosmographer - This Persian cosmographer⁶⁴² flourished under Ṭughril II (last Saljūq ruler of 'Irāq and Kurdistan, 1177-1194). In 1160 al-ṬŪsĪ wrote a Persian cosmography entitled "*Aḡā'ib al-makhlūqāt*" (The Marvels of Creatures).

The unique manuscript of this work (Gotha Persicus 35) contains at the beginning six very crude maps representing the Caspian Sea, the Mediterranean Sea, Jibāl, Sind, the Arabian Sea, and Arabia.

191140

ṬŪsĪ, Sharafeddin

ṬŪSĪ (Sharaf al-Dīn al-ṬŪsĪ)

Sharaf al-Dīn al-Muzaffar Ibn Muḥammad Ibn al-Muzaffar al-ṬŪsĪ
b. in ṬŪs (Iran); d. in Iran in 1213 A.D

Astronomer, mathematician - Nothing is known about the first years of al-ṬŪsĪ's life^{643,644}; but it is reported that, faithful to the tradition of medieval scholars, he went on a long journey to some of the major cities of the time. He taught at Damascus, probably about 1165. His most distinguished student there was Abu'l Fadl (b. 1135), an excellent carpenter who helped make the wood paneling of the Bīmāristān al-Nūrī (1154-1159) before discovering the joys of Euclid and Ptolemy. Al-ṬŪsĪ most probably stayed in Aleppo, where one of his pupils was a respected member of the city's Jewish community, Abu'l Fadl Binyāmīn (d. 1207/1208), whom he instructed in the science of numbers, the use of the astronomical tables, and astrology, and in a less advanced level, in the other sciences.

Al-ṬŪsĪ's most outstanding pupil, however, was Kamāl al-Dīn Ibn Yūnus (d. 1243) of Mūṣul, through whom al-ṬŪsĪ's teachings passed to Naṣir al-Dīn al-ṬŪsĪ (d. 1274) and Athīr al-Dīn al-

⁶⁴² Sarton, p. 413

⁶⁴³ Anbouba, pp. 514-517

⁶⁴⁴ Sarton, pp. 622-623

ZEITSCHRIFT FÜR GESCHICHTE
DER ARABISCH-ISLAMISCHEN
WISSENSCHAFTEN

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Fuat Sezgin

in Zusammenarbeit mit
M. Amawi, C. Ehrig-Eggert,
E. Neubauer

— Mozaffer et-Tūsi

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1989

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an der Johann Wolfgang Goethe-Universität
Frankfurt am Main

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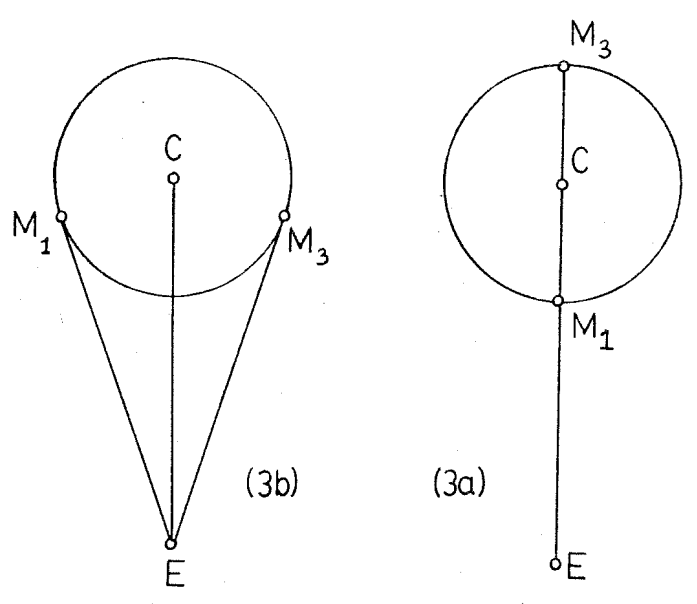


Figure 2

M_3 are in diametrically opposite positions on the epicycle, M_2 and M_4 are also diametrically opposite (because of (4)), so that c_2 and c_4 must both be zero or have opposite sign. We conclude $c_2 = c_4 = 0$. So if we know that M_2 is not diametrically opposite M_1 and M_4 is not diametrically opposite M_3 (as stated in (3a)), then we must conclude $M_1 = M_2$ and $M_3 = M_4$, so that $\alpha_1 = \alpha_2$ and $\alpha_3 = \alpha_4$ as desired.

For (3b), c_1 and c_3 are the maximal and minimal possible values of the lunar equation, so that $c_1 \geq c_2$ and $c_3 \leq c_4$. Therefore by (5) $c_1 = c_2$ and $c_3 = c_4$, and thus $M_1 = M_2$ and $M_3 = M_4$.

Thābit did not give a detailed description involving figures such as Figure 2, but his motivation of the conditions (3a) and (3b) is essentially correct and sheds light on a difficult passage in the *Almagest*.

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JAN P. HOGENDIJK*

R. Rashed (Ed., Transl.), Sharaf al-Dīn al-Tūsī; *Œuvres mathématiques. Algèbre et géométrie au XII^e siècle*. Paris: Société d'édition „Les Belles Lettres”, 1986. Vol. 1: 470 pp., ISBN 2-251-35562-6. Vol. 2: 459 pp., ISBN 2-251-35563-4.

These two volumes contain editions and French translations of the three extant mathematical works of Sharaf al-Dīn al-Tūsī (who flourished in the end of the 6th century H./12th century A. D.): a brief, hitherto unpublished, treatise about the asymptotes of the hyperbola, a short treatise on elementary geometry that had been studied previously by Suter,¹ and a very long hitherto unpublished text whose title has been lost, but to which we will refer as the *Algebra*. This text is one of the most important mathematical works from the Arabic-Islamic tradition, because it contains the most profound medieval discussion of cubic equations that is known to be extant.

The Arabic-Islamic mathematicians appear not to have known the algebraical solution of the cubic equation. However, in the fourth/tenth century they constructed the roots of various cubic equations geometrically by means of conic sections. In the 11th century, ‘Umar al-Khayyām wrote his famous *Algebra*, in which he gave geometrical constructions by means of conic sections of the roots of all types of cubic equations. (Because the medieval Arabic-Islamic mathematics only worked with positive coefficients, they had to distinguish 17 types of cubic equations.) ‘Umar al-Khayyām’s treatment was incomplete in various respects. Some types of equations, such as $x^3 + c = ax^2 + bx$, do not have roots for all possible choices of the coefficients. In such

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¹ H. Suter, Einige geometrische Aufgaben bei arabischen Mathematikern. *Bibliotheca Mathematica*, 3. Folge, 8 (1907-8), pp. 23-36, reprinted in: H. Suter, *Beiträge zur Geschichte der Mathematik und Astronomie der Araber*, ed. F. Sezgin, Frankfurt (Institut für Geschichte der Arabisch-Islamischen Wissenschaften) 1986, vol. 2, pp. 217-230.



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سلسلة تاريخ العلوم العربية (٥)

الجبر والهندسة

في القرن الثاني عشر

مؤلفات شرف الدين الطوسي

الدكتور رشدي راشد

ترجمة: الدكتور نقولا فارس

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Sharaf al-Dīn al-Ṭūsī
Oeuvres mathématiques
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Al-Birūnī était né en 973, le livre sur « Les clefs de l'astronomie » est l'une de ses œuvres de jeunesse car il paraît avoir été composé entre 994 et 1004 (cf. pp. 68-69). Selon son habitude, cet auteur fait le bilan de ce qui l'a précédé, il cite soigneusement ses sources, en particulier les œuvres de ses maîtres, puis présente ses propres solutions aux problèmes posés. Après la dédicace et l'introduction générale, son livre est très nettement divisé en quatre parties :

- 1) Histoire de la découverte de la figure « qui dispense du quadrilatère complet », et détail du rôle des prédécesseurs immédiats d'al-Birūnī, qui rend à chacun ce qui lui revient (p. 96-103).
- 2) Reprise critique des démonstrations antérieures, avec proposition d'autres solutions (p. 104-151).
- 3) Systématisation, par l'auteur, de sa propre solution, avec une classification des différents triangles sphériques en dix catégories selon la qualité de leurs angles (obtus, aigus ou droits), et résolution des quinze cas déterminables par le groupement deux par deux des éléments connus (côté ou angles) (p. 152-195).
- 4) Application des raisonnements précédents à la solution de vingt-quatre problèmes classiques d'astronomie, avec rappel des méthodes antérieures (p. 196-297).

Cette dernière partie couvre environ la moitié du livre, et toutes les démonstrations antérieures sont ainsi orientées vers ces applications, mais l'intérêt de l'ouvrage réside surtout dans les trois premières parties, purement trigonométriques.

Cette œuvre d'al-Birūnī n'avait encore jamais été éditée; elle avait fait l'objet d'une présentation par E.S. Kennedy, et le manuscrit qui la contient avait simplement été reproduit photographiquement en Iran, ce qui ne permet pas un véritable accès au texte, d'autant plus que l'écriture de ce manuscrit unique est entièrement dépourvue de points diacritiques. Le texte arabe et sa traduction sont présentés face à face en édition bilingue, et le contenu en est longuement présenté et commenté : M.T.D. commence par situer et expliciter les raisonnements cités par al-Birūnī, c'est-à-dire ceux de ses maîtres ou de ses contemporains, Abū 'l-Wafā', Abū Naṣr, al-Ḥuḡandī, Kušyār b. Labbān; elle passe ensuite à la période précédente en donnant l'évolution de la trigonométrie sphérique au IX^e siècle à partir du double héritage grec et indien, et en s'attardant longuement sur l'œuvre de Ḥabaš, dont la partie trigonométrique est ici dépouillée de façon systématique pour prouver que c'est cet auteur qui a fixé le cadre dans lequel cette science s'est développée par la suite; enfin, elle détaille les éléments originaux dans les « Clefs de l'astronomie » en cherchant quelle a été leur influence sur le « Traité du quadrilatère » de Naṣir al-Dīn al-Ṭūsī. Dans ces chapitres introductifs, M.T.D., dans la majorité des cas, s'appuie sur des œuvres existant encore seulement sous forme manuscrite, et leur dépouillement est très précieux. Ajoutons que les index de vocabulaire et de calculs seront appréciés par tous les historiens des sciences arabes.

On pourrait faire à la présentation de l'ouvrage quelques critiques, telles que la seule mention de la page pour un terme donné en index, ou la difficulté de trouver le sens de certains symboles utilisés pour caractériser des formules. Mais il ne s'agit là que de points de détail, et cette

publication constituera désormais un ouvrage de référence pour l'étude de la trigonométrie sphérique dans le monde arabe aux IX^e et X^e siècles.

Régis MORELON
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Muraffa et Tusi

Sharaf al-Dīn AL-ṬŪSĪ, *Œuvres mathématiques* (Algèbre et Géométrie au XII^e siècle),
texte établi et traduit par Roshdi Rashed. Paris, Les Belles Lettres, 1986. 2 tomes,
16 × 24 cm., 470 et 450 p.

Šaraf al-Dīn al-Ṭūsī est un scientifique de la fin du XII^e siècle. On sait peu de chose à son sujet, puisque son nom n'est mentionné par les biobibliographes arabes que lorsqu'ils signalent que tel ou tel auteur avait travaillé sous sa direction à Damas ou à Mossoul vers la fin du XII^e siècle (cf. tome I, p. XXXII-XXXVI). Cet auteur est surtout connu pour son « astrolabe linéaire », ou « bâton d'al-Ṭūsī », sur lequel plusieurs publications avaient été faites depuis la fin du XIX^e siècle. Rien n'avait été publié sur son grand traité d'algèbre, avant les articles de R. Rashed, à partir de 1974 (articles repris dans *Entre arithmétique et algèbre*, Paris 1984, p. 93-194).

Les deux tomes présentés ici regroupent les œuvres mathématiques de cet auteur qui nous sont parvenues : le grand traité sur les équations (244 pages d'arabe), puis un opuscule sur l'asymptote et une épître sur un problème de construction géométrique (respectivement 6 et 7 pages de texte arabe). C'est évidemment le « traité sur les équations » qui est la pièce maîtresse de cet ensemble. Cette œuvre importante était connue dans un manuscrit unique de la fin du XVIII^e siècle, conservé à Londres, jusqu'à la découverte par R.R. du modèle de celui-ci, à Patna en Inde, manuscrit de la fin du XIII^e siècle, dans lequel manque le premier tiers du texte; puis un troisième manuscrit partiel tardif, à Venise, contient le premier cinquième du traité. Ces trois témoins permettent dans de bonnes conditions l'établissement de la version de ce texte à laquelle tous trois donnent accès.

Ce travail de Šaraf al-Dīn est présenté par R.R. comme l'œuvre la plus importante, avec celle d'al-Ḥayyām, entre les travaux d'Apollonius et ceux de Descartes et de Fermat. En effet, après l'apparition de l'algèbre avec al-Ḥwārizmī au IX^e siècle, puis le développement de cette science au X^e siècle, c'est al-Ḥayyām, au XI^e, qui systématise le couplage entre algèbre et géométrie en faisant une étude des équations du 3^e degré et en trouvant leur solution par l'intersection de deux coniques. Cette formulation théorique du problème de la construction géométrique des équations algébriques paraissait totalement isolée dans le monde arabe, dans la mesure où aucun autre auteur ne semblait avoir poursuivi le travail. Or la mise à jour de l'œuvre présentée ici montre qu'al-Ḥayyām avait créé une tradition et que Šaraf al-Dīn a prolongé et développé largement les analyses de son prédécesseur, environ un siècle plus tard.

Šaraf al-Dīn étudie d'abord la parabole et l'hyperbole, l'étude du cercle étant supposée connue, dans la mesure où il aura besoin de tout cela dans la suite de son traité. Puis il classe les 25 équations qu'il retient en deux grandes catégories : celles qui admettent des solutions positives, et celles qui présentent des « cas impossibles » car elles admettraient des solutions négatives. La

SHARAF AL-DĪN AL-ṬŪSĪ Little is known about the life of Sharaf al-Dīn al-Muzaffar ibn Muḥammad ibn al-Muzaffar al-Ṭūsī. He was born in Ṭūs (Iran), probably around 1135. He spent part of his life in Syria and Iraq, and died in Iran around 1213. He had a number of pupils, including the famous polymath Kamāl al-Dīn ibn Yūnus.

Sharaf al-Dīn al-Ṭūsī was one of the best mathematicians of the Arabic tradition. His most important work is a treatise on cubic equations, in which he develops the theory quite beyond the limits reached by al-Khayyām. The Arabic mathematicians only considered positive coefficients, and they therefore distinguished various types of cubic equations, such as $x^3 + bx = c$, $x^3 + c = bx$, $x^3 = bx + c$, $x^3 + ax^2 = bx + c$, $x^3 + bx = ax^2 + c$ and so on. For all these types, ‘Umar al-Khayyām had indicated a geometrical solution of a positive root x (negative and zero roots were not taken into account). For al-Khayyām, the coefficients a , b , c were not in the first place numbers, but a is a line segment, b is a rectangle, and c is a parallelepiped. Using these data, al-Khayyām constructed two conic sections, and he supposed that these conics intersected in a point P. He then constructed x by means of P. It is obvious that this procedure only works if point P exists. For some types of cubic equations (for example $x^3 + bx = c$ and $x^3 = bx + c$) it is easily proven that a point P always exists, so the equation always has a root x . For other types of cubic equations (such as $x^3 + c = bx$) P may or may not exist, depending on the coefficients of the equation. The only way in which al-Khayyām could find out whether point P existed was by actually drawing the conic sections, which could be done with limited accuracy (it is unclear whether al-Khayyām ever constructed a root x this way, because the subject seems to have been of theoretical interest only). Thus it was not precisely clear when a cubic equation like $x^3 + c = bx$ had a root.

For the particular case of $x^3 + c = ax^2$, some predecessors of al-Khayyām had shown that a root x exists if and only if $c \leq (4/27)a^3$. Thus these mathematicians knew in advance whether a root existed, and they only needed the conic sections to construct x . Sharaf al-Dīn showed that similar (but much more complicated) conditions exist for all types of cubic equations which do not always have positive roots. He meticulously proved the correctness of these conditions by means of the methods of “geometrical algebra” used by Euclid in Book II of the *Elements*. He then gave a geometrical construction of the roots x , and (unlike al-Khayyām) a numerical procedure, which could be used to approximate the roots with any desired accuracy. In the theory of cubic equations, no further progress was made until the discovery of the algebraical solution in Italy after 1500.

In addition to his treatise on equations, Sharaf al-Dīn wrote two short works on elementary geometry. He also in-

vented the “linear astrolabe”, consisting of a staff with a plumb line and two cords. The staff contained the markings of the North–South line of an ordinary astrolabe. Although this instrument is easier to construct than an ordinary astrolabe, it does not look nice and it is not user-friendly. Thus it is not surprising that no examples have survived.

JAN P. HOGENDIJK

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See also: ‘Umar al-Khayyām

SHEN GUA The Chinese scholar, statesman, and scientist Shen Gua was born in 1031 in present-day Hangzhou (Zhejiang). Born into a family of gentry, he entered civil service in 1054, holding minor posts in different provinces where he distinguished himself in a series of projects involving water control. After passing the national doctoral examinations in 1063 he moved to the capital, Kaifeng, and in 1072 was named Director of the Astronomy Bureau. In that position he reorganised the Bureau, planned a program of astronomical observations, and devised a calendar that was effective from 1075 to 1092. His work as astronomer alternated with missions to the provinces, where he was again able to display his skills in hydraulic engineering. The relief maps he made in 1074 (using wooden plates, sawdust impregnated with glue, and melted wax) are among the most celebrated specimens of Chinese cartography.

The whole of Shen’s career was influenced by his partnership with Wang Anshi (1021–1086), the leader of the “New Policies” (*xinfa*) group which advocated a reform program aimed at strengthening the central government, regulating finances by law, and exploiting nature for the state’s benefit. Shen’s memorials to Emperor Shenzong (r. 1067–1084) on a variety of subjects — including military strategy and tax levies — won approbation in a dozen instances. In a related development, Shen was sent as an envoy to the Khitan court in 1075, and, in a feat of diplomacy, was able to put at least a temporary end to the borderland skirmishes and the repeated Khitan claims of Chinese land.

7. Conclusion

Bartoli's thought experiment, as reworked by Boltzmann, was important in two areas of modern physics: black-body theory and radiation pressure. Bartoli was probably the first to refer to the energy density of black-body radiation in connection with heat trapped in a cavity. In 1893, Wilhelm Wien derived his displacement law by extending Boltzmann's reasoning to cover the separate wavelengths of black-body radiation. Wien's result, an important milestone on the road to Planck's theory, was based both on Boltzmann's works of 1884 and a modified version of the Bartoli cycle.³⁴ At that time, light pressure was still only hypothetical, but in 1901 Peter Lebedew demonstrated the existence of the 'Maxwell-Bartoli pressure' experimentally.³⁵ Lebedew's point of departure was the same as Wien's, namely the research of Bartoli and Boltzmann.

From the works of Boltzmann, Wien and Lebedew, as well as from later historical comments, Bartoli would appear to be one of the pioneers of light pressure. But he never stated that this sort of pressure exists, and after 1884 he abandoned the research convinced that the hypothesis of light pressure was incorrect. The literature perpetuates a general misunderstanding whereby Bartoli argued that light must exert pressure in order not to violate the second law.³⁶ On the contrary, he never claimed to have deduced the necessity of light pressure; furthermore, it is evident from the thought experiment that no such one-to-one correspondence exists. All that one can conclude, and all that Bartoli did conclude, is that the hypothesis of light pressure would save the second law which does not imply that the law can be saved *only* by accepting this hypothesis, or that light pressure actually exists.

³⁴ W. Wien, 'Eine Neue Beziehung der Strahlung Schwarzer Körper zum Zweiten Hauptsatz der Wärmetheorie', *Sitzungsberichte der Königlich-Preussischen Akademie die Wissenschaften (Berlin)*, 1893, 55-62. For the development of black-body theory, see Hans Kangro, *Early History of Planck's Radiation Law* (London, 1976), especially pp. 90-148.

³⁵ P. Lebedew, 'Die Druckkräfte des Lichtes', *Annalen der Physik*, 6 (1901), 433-58.

³⁶ E.g. Mehra and Rechenberg (footnote 1), p. 29; Worall (footnote 2), p. 154; Schagrin (footnote 2), p. 939; Brush (footnote 21), p. 539; Whittaker (footnote 31), p. 275; M. Jammer, *The Conceptual Development of Quantum Mechanics* (New York, 1966), p. 7.

Essay Review

The Interpretation of the Algebra of Sharaf al-Dīn al-Ṭūsī

SHARAF AL-DĪN AL-ṬŪSĪ, *Oeuvres mathématiques*. Edited, translated into French and commented by R. Rashed. Paris, Société d'édition Les Belles Lettres, 1986, 2 vols, CXCI + 129 + XXXV pp. CXLI + 165 pp. F640-00.

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These two volumes contain all the known works of this mathematician of the second half of the twelfth century: a *Treatise on the Theory of Equations* (*al-mu'ādalāt*) by far the most important; one small book of five pages *On the Two Lines which approach one another but do not meet* (asymptotes) and a seven-page *Letter on a Problem of Geometrical Construction* relating to the construction in a given square of a rectangle and three trapeziums, all related to each other by a given ratio. I shall only consider in this review the treatise on equations, the whole of which is edited and translated here for the first time.

Volume one contains an Introduction in which Rashed ascribes to al-Ṭūsī's theory of equations its place in the history of Arabic algebra. He recalls how after al-Khwarizmi, the development of algebra has allowed Arabic mathematicians to translate algebraic problems into geometry and vice versa. He then asserts that such a possibility became a 'projet scientifique' with al-Khayyām (1048-1131), who devoted the main part of his book on *Algebra* to the geometrical solutions of the cubic equations by means of conic sections. The *Treatise* of al-Ṭūsī goes much farther than Khayyām's, not only by the number and forms of the equations treated but also by the nature of these equations and the way by which they were solved.

This *Treatise* is composed of two different parts, to which Rashed devotes the two volumes respectively.

The first part contains six binomial equations of degree ≤ 3 ; six trinomial cubic equations reducible to quadratic equations and eight cubic equations which can always have (positive) solutions. They are preceded by four lemmas and five geometrical constructions, all concerning the hyperbola and parabola, by means of which al-Ṭūsī solves the cubic equations of this first part. Al-Ṭūsī also gives the numerical approximated solutions by a method analogous to that of Ruffini-Horner. The equations of this first part are numbered by Rashed from 1 to 20.

The second part of the *Treatise* contains five fundamental types of cubic equations—numbered by Rashed from 21 to 25—and which distinguish themselves from the last eight of the first part by the fact that for some values of the coefficients they are 'impossible', i.e. they have no (positive) solutions. In the solutions of these equations, al-Ṭūsī does not use conic sections and Rashed asserts (Vol. I, pp. XXII, XXIII) that the method of al-Ṭūsī is that of finding the maximum of a polynomial function by means of the derivative of that function. He even claims that 'la présence de la notion de dérivée n'est ni fortuite, ni secondaire, mais, au contraire intentionnelle'.

Aristotle was careful to restrict the scope of moral investigation to propositions that do not satisfy his canons of scientific certainty but are nevertheless generally accepted (*endoxa*). Aristotle's medieval followers, however, chose to treat the matter as an open question; perhaps the link between religious belief and ethical norms, coupled with the awareness of other religious systems, made the appeal to "generally accepted" propositions unsatisfactory. Whatever the cause, the scientific status of ethical claims was the target of intense research and debate throughout the Middle Ages.

Krieger's work is a full-length study of the position taken on this matter by the fourteenth-century Parisian master Jean Buridan. In many ways Buridan is an excellent choice. His views were enormously influential on his own and succeeding generations (his *Questions on the "Nicomachean Ethics"* was printed as late as 1610 at Oxford, and read into the beginning of the eighteenth century). As a secular member of the Arts Faculty, Buridan had no particular axe to grind about the relative status of intellect and will, a question hotly disputed among the different religious orders. Finally, on a more mundane level, Buridan's ethical views have not yet been subjected to a systematic study; the recent articles by J. J. Walsh and the studies of J. B. Korolec, unlike Krieger's book, have been directed to particular aspects of Buridan's views.

Krieger shows a solid familiarity with Buridan's texts, referring most often to the *Questions on the "Nicomachean Ethics,"* but including discussions of the *Questions on the "Metaphysics"* and the *Questions on the "Physics"* as well. Unfortunately he does not discuss Buridan's *Questions on the "De anima,"* which is relevant for understanding Buridan's conception of knowledge as an acquired disposition and for the possibility of knowledge of singulars and particular events. Krieger refers to the views of other philosophers in the course of his discussion, most notably Aristotle, Thomas Aquinas, and William of Ockham, but he primarily concentrates on Buridan. While this is perfectly sensible, it might have been illuminating to examine Buridan's allegiance to Seneca (proclaimed at the beginning of the *Questions on the "Nicomachean Ethics"*). Still, the philosophers considered are hardly out of place, and

Krieger's treatment is fair and balanced.

There are many ways into the topic; Krieger focuses on practical reason, which plays a crucial role in ethics: it is the aspect of the intellect involved in deliberation and choice, the application of the cardinal virtue of prudence, the faculty that restrains the appetite. Yet the role of practical reason is not restricted to ethics; it also is that faculty by which general scientific principles are applied to particular cases and may also serve as the ultimate grounding of "indemonstrable" principles of knowledge, such as the principle of noncontradiction (which Krieger discusses in Ch. 2.3). In short, practical reason is well suited to provide a bridge between unquestioned examples of scientific knowledge and the disputed case of ethical knowledge.

The main outlines of Krieger's interpretation of Buridan might be represented as follows. Practical reason supplies the intellect with particular minor premises to be used in drawing conclusions; in the case of demonstrative science, the result is an application, or a concrete case of knowledge of the case at hand; in the case of ethics, the result is a practical syllogism, which culminates not merely in a conclusion but in an action. Both applied science and the practical syllogism are equally forms of knowledge, on a par with the purely theoretical knowledge obtained through a demonstrative syllogism with no particular premises. The certainty of the particular minor premises involved in such syllogisms is guaranteed either through what other philosophers call "intuitive cognition" or by practical wisdom (depending on the application of practical reason). Indeed, the foundations of all theoretical reasoning depend on practical reason, as noted above, and so all forms of knowledge may be as "scientific," in a sense, as physics or metaphysics. There can be a "science of the virtues" since the moral virtues are interestingly subordinate to reason; the perfection of reason is that which leads to complete happiness or felicity. (Technically, happiness is an operation according to complete virtue, which is only possible through the perfection of reason.) Since knowledge of all kinds is an acquired mental disposition (*habitus*), the conditioning necessary for correct moral behavior is of a piece with the conditioning necessary for drawing proper theoretical inferences. To clarify this point, Krieger elaborates the notion of

normative practical knowledge (*normativ-praktische Wissenschaft*), sketching it by contrast with purely theoretical knowledge on the one hand and the so-called mechanical arts on the other.

Krieger develops this outline through a series of comparisons, discussion of texts, and thematic treatment. His discussion is generally good, although not always as incisive or sharp as it could be. I would have appreciated more discussion of the mechanical arts than that largely confined to pages 277–281 and of the contrast or continuity Krieger finds with moral science. Nor does he try to connect his discussion with modern views of science. But these are the shortcomings of a pioneer; Krieger has attempted a systematic study of the views of one of the best minds of the Middle Ages, and that is a sufficiently difficult and important project on its own. Those who work in the history of science will find Krieger's work a stimulating and fertile ground for exploring a scientific paradigm flexible enough to be extended to cover ethical and normative claims. Those who have an interest in the history of ethics as well will find it a significant contribution.

However, before closing, I must register an objection: Krieger's book, which is carefully detailed and researched, unfortunately does not have an index. This does not lessen the value of the contribution, but makes it less useful as a tool of scholarly research.

PETER KING

Sharaf al-Din al-Tusi. *Oeuvres mathématiques: Algèbre et géométrie au XIIe siècle.* Edited and translated by Roshdi Rashed. (Collection Sciences et Philosophie Arabes, Textes et Études.) 2 volumes. 900 pp., figs., bibl., index. Paris: Société d'Édition "Les Belles Lettres," 1986. Fr 640.

Those working in the history of Arabic or medieval science have commonly assumed that most technical problems occurring in medieval mathematical texts could be solved with mathematical principles and techniques learned at the high school level. For the works of Sharaf al-Din al-Tūsī, however, this generalization is clearly inaccurate, because this twelfth-century (fl. 1180) mathematician stands brilliantly outside the main confines of the Arabic tradition.

The field of algebra, first formulated as a

discipline within the Arabic tradition from which al-Tūsī draws, was in its early stages not so much a theory of equations as a systematic procedure for solving a coherently classified set of problems translatable into equations, mainly of the second or the third degree. When people read the works of al-Khwārizmī (fl. 830), al-Karajī (10–11th century), and al-Samaw²al (d.ca.1174), they can easily detect progress toward a systematic standardization of the solutions of algebraic equations, which remained restricted mainly to the domain of the second-degree equations. Only with al-Khayyām (d.ca.1131), whose work was also recently edited and translated by Rashed (Aleppo, 1981), does one feel that the Arabic algebraic tradition was coming of age, that the serious study of algebraic equations was becoming an independent enterprise whose object was the equations themselves and the conditions under which they have real solutions. As the sophistication of algebraic equations increased to the third degree level, al-Khayyām resorted to solutions employing the intersection of conic sections and thus inaugurated a new line of investigation in Arabic algebra, namely, the application of geometric techniques to algebraic equations, or the "geometrization" rather than the "arithmetization" of algebra practiced by the school of al-Karajī and al-Samaw²al.

In the works of Sharaf al-Din al-Tūsī we have yet another departure. The application of geometric techniques to solve algebraic equations was no longer sufficient, nor were the numerical techniques developed by the other algebraists. The requirement, as perceived by al-Tūsī, was not to solve a specific algebraic equation or type of algebraic equation, but to study the conditions under which certain equations would have solutions (i.e., to determine the existence of the roots themselves) and to classify these equations systematically in all their possible combinations. By permutation, third-degree equations yield twenty-five types. Al-Khayyām had already enumerated them, solved all those that could be solved by numerical means, and applied the geometry of the conic sections to the others. Furthermore, he had actually hinted that this last group of equations—those solved by geometric means—might one day be solved by algebraic means. Al-Tūsī responded to this challenge by writing the work now known to us as *The Equations*,

INNOVATION AND TRADITION IN SHARAF AL-DĪN
AL-ṬŪSĪ'S *AL-MU'ĀDALĀT**

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The largest treatise in Roshdi Rashed's edition of the mathematical works of Sharaf al-Dīn al-Ṭūsī is *Al-mu'ādalāt*, a work devoted to the solution of cubic equations. This treatise shows that Islamic authors went considerably beyond the achievements of 'Umar al-Khayyāmī in three areas: (1) finding conditions for the existence of solutions to cubic equations, (2) discovering algorithms for calculating these solutions, and (3) justifying these algorithms. Although so much is clear, it is still no easy task to understand Sharaf al-Dīn's thought processes and so achieve an understanding of the historical filiations of the document. Rashed has argued that Sharaf al-Dīn discovered the derivative of cubic polynomials and realized its significance for investigating conditions under which cubic equations were solvable; however, other scholars have suggested quite different explanations of Sharaf al-Dīn's thinking, which connect it with mathematics found in Euclid or Archimedes. Our purpose in the present essay review is to decide which of these three readings seems best to fit the text.

HISTORIANS OF SCIENCE, NO LESS than scientists themselves, are accustomed to seeing their discipline change slowly. Most scholars spend their lifetimes making small changes or additions, and their satisfaction comes from knowing that their work, if done well, will be something that the next generation can build upon. Rarely does a historian find a document that forces major changes of fundamental ideas about a given field, and for that reason there was both excitement and caution when Prof. Rashed announced in 1974 his discovery of a document that would compel us to revise substantially our history of Muslim achievements in the theory of the solution of algebraic equations.

The history of the theory of equations has already provided several instances of discoveries of source material resulting in major changes in our understanding of the history of mathematics, an outstanding instance being the decipherment of the mathematical cuneiform texts. These showed that the Babylonians of Hammurabi's time knew both the exact solution to the general equation of degree 2 and good numerical methods to solve the equation $x^2 = m$.

* This is a review article of: *Sharaf al-Dīn al-Ṭūsī, oeuvres mathématiques: Algèbre et géométrie au XII^e siècle*. Edited and translated by ROSHDI RASHED. 2 vols. Collection Sciences et Philosophie Arabes: Textes et Études. Paris: SOCIÉTÉ D'ÉDITION "LES BELLES LETTRES," 1986. Pp. cxci + 129 + xxvi (Arabic summary), and cxliii + 166. FF 640.

Another instance is Fr. Woepcke's discovery in 1851 of Omar Khayyam's theory of cubic equations based on the theory of the conic sections. Woepcke's discovery, together with the extracts that he presented from the other works of the 10th-11th centuries, made it clear that not only had Omar Khayyam found geometric solutions to all possible cubic equations, but also that his work was part of a major current of Islamic work on problems whose origins lay in such classical problems as duplicating the cube and trisecting an arbitrary angle.

Rashed neatly characterizes this work as involving a "double-translation," first of a geometric problem (such as the ones referred to above) into the algebraic problem of solving a cubic equation and then of that algebraic problem into the geometric problem of intersecting conic sections. Thus, after the second translation one is back to a geometric problem, but one of a standard type and very different from the initial problem.

A third major discovery in this field was that of P. Luckey, about a century after Woepcke's publication of Khayyam's *Algebra*, who showed that by the time of al-Kāshī, Muslim mathematicians had not only discovered an efficient numerical method for finding roots of equations, known today as the Ruffini-Horner method, but they had also developed the full theory of decimal fractions.

There were, however, some gaps in the story as it had evolved thus far. For example, Omar Khayyam's

-Tūsī, Şerefeddīn

INNOVATION AND TRADITION IN SHARAF AL-DĪN
AL-TŪSĪ'S *AL-MU'ĀDALĀT**

J. L. BERGGREN

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23 HAZ 2010

The largest treatise in Roshdi Rashed's edition of the mathematical works of Sharaf al-Dīn al-Tūsī is *Al-mu'ādalāt*, a work devoted to the solution of cubic equations. This treatise shows that Islamic authors went considerably beyond the achievements of 'Umar al-Khayyāmī in three areas: (1) finding conditions for the existence of solutions to cubic equations, (2) discovering algorithms for calculating these solutions, and (3) justifying these algorithms. Although so much is clear, it is still no easy task to understand Sharaf al-Dīn's thought processes and so achieve an understanding of the historical filiations of the document. Rashed has argued that Sharaf al-Dīn discovered the derivative of cubic polynomials and realized its significance for investigating conditions under which cubic equations were solvable; however, other scholars have suggested quite different explanations of Sharaf al-Dīn's thinking, which connect it with mathematics found in Euclid or Archimedes. Our purpose in the present essay review is to decide which of these three readings seems best to fit the text.

HISTORIANS OF SCIENCE, NO LESS than scientists themselves, are accustomed to seeing their discipline change slowly. Most scholars spend their lifetimes making small changes or additions, and their satisfaction comes from knowing that their work, if done well, will be something that the next generation can build upon. Rarely does a historian find a document that forces major changes of fundamental ideas about a given field, and for that reason there was both excitement and caution when Prof. Rashed announced in 1974 his discovery of a document that would compel us to revise substantially our history of Muslim achievements in the theory of the solution of algebraic equations.

The history of the theory of equations has already provided several instances of discoveries of source material resulting in major changes in our understanding of the history of mathematics, an outstanding instance being the decipherment of the mathematical cuneiform texts. These showed that the Babylonians of Hammurabi's time knew both the exact solution to the general equation of degree 2 and good numerical methods to solve the equation $x^2 = m$.

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quality is brought into the sea from below. Finally, the *De'ot ha-Pilosofim* succeeds in solving a question that was left open in the *Otot ha-Shamayim*: the nature of the relationship between the dry exhalation and the earthy substance. By advisedly and creatively combining Ibn Rushd's commentaries, Ibn Falaqera succeeds in pointing out that the dry exhalation is of earthy origin, since it is the residue of combustion, but that earthy substances, in and of themselves, play only a secondary role in producing saltiness.



09 OCAK 1996

LE CALCUL DU MAXIMUM ET LA "DÉRIVÉE" SELON SHARAF AL-DĪN AL-ṬŪSĪ

NICOLAS FARÈS

0. INTRODUCTION

Le présent article se propose d'analyser les moyens et les techniques utilisés par Sharaf al-Dīn al-Ṭūsī, lors de son calcul du maximum d'une fonction polynomiale du troisième degré $f(x)$, où apparaît systématiquement une équation équivalente à $f'(x) = 0$.¹ Il ne s'agit pas de revenir sur l'histoire de ces notions mathématiques qui ont été étudiées dans l'ouvrage que R. Rashed a consacré aux travaux mathématiques d'al-Ṭūsī.² Dans cet ouvrage, l'auteur regroupe les œuvres mathématiques de Sharaf al-Dīn al-Ṭūsī, qu'il établit, traduit et commente. Son Introduction constitue, à elle seule, une étude approfondie de ces œuvres, à la fois sur le plan mathématique et sur le plan historique.³ Il y met en lumière, en particulier, la présence de l'expression de la dérivée d'une fonction polynomiale du troisième degré et souligne la fréquence de son utilisation à travers le *Traité des équations* d'al-Ṭūsī. Il met l'accent sur le rôle que joue l'expression de la dérivée dans l'étude et le calcul du maximum qu'al-Ṭūsī effectue pour des fonctions polynomiales du troisième degré et signale sa présence dans des parties du *Traité* consacrées aux résolutions numériques. De plus, Rashed n'omet

¹ Il s'agit, en fait, de l'équation $\frac{1}{3}f'(x) = 0$.

² *Sharaf al-Din al-Ṭūsī: Œuvres mathématiques. Algèbre et géométrie au XII^e siècle*, éd. R. Rashed, Collection Sciences et Philosophie Arabes. Textes et Études, 2 vol. (Paris, 1986). Traduit en arabe par N. Farès, Centre for Arab Unity Studies (Beyrouth, 1995), à paraître. Voir aussi R. Rashed, "Résolution des équations numériques et algèbre. Sharaf al-Din al-Ṭūsī, Viète," *Archive for History of Exact Sciences*, 12, 3 (1974): 244-90; repris dans Id., *Entre arithmétique et algèbre: Recherches sur l'histoire des mathématiques arabes* (Paris, 1984), Chap. 3, pp. 148-93.

³ Dans C. Houzel, "Sharaf al-Din al-Ṭūsī. Œuvres mathématiques. Algèbre et géométrie au XII^e siècle," *Gazette des mathématiciens*, 39 (1989): 58-63, on trouve un compte-rendu du livre du même titre.

A PARAÎTRE:

Thābit ibn Qurra: *Œuvres astronomiques*, texte établi et traduit par R. Morelon.

Al-Kindī: *Œuvres optiques*, texte établi et traduit par R. Rashed et H. Hugonnard-Roche, J. Jolivet, H. Sinaceur.

Ibn al-Haytham: *Œuvres mathématiques*, texte établi et traduit par R. Rashed.

SHARAF AL-DĪN AL-ṬŪSĪ
ŒUVRES MATHÉMATIQUES

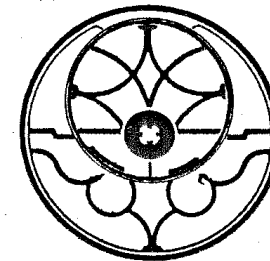
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Tome II

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541. SHARAF AL-DİN AL-TUSİ

Sharaf al-Dīn al-Muzaffar ibn Muḥammad al-Ṭūsī (d. 1213), from Tus, worked in Hamadhan. mathematician and astronomer, teacher of Kamāl al-Dīn Musā ibn Yūnis (No 576).

See: GAL (I 622), GAL² (I 858-859), GAS (V 399), IHS (II 622-623), KWA (II 133, 185), KWA² (III 470, 481), KZ (VI 386), MA (170-171). MAA (134), MAMS (II 365-367), SSM (149); Anbuba [4] (DSB), Hogendijk [36] (ENWC), Rashed [16, 26], Tuqan [1] (406).

M1. Problems of Algebra and Almucabala (Masāil al-jabr wa'l-muqābala) = Treatise on Equations (Risāla <fi>'l-mu'adalāt) - manuscript of an abridged revision (talkhīṣ wa tahdhīb) - London (Ind. 767/3). Edition and French translation: Sh. al-Ṭūsī [1].

Research: Amir-Moēz and Chavoshi [1], Berggren [12], Hogendijk [19], Houzel [2-3], Krasnova and Tagi-zade [1], Parès [1], Rashed [9]. Exposition of an approximate solution of cubic equation near solutions of Viète and Newton and a generalization of the extraction of cubic roots in the works of Ibn Labbān (No 308, M1) and al-Nasawī (No 341, M1). The existence of real roots, both geometric (similar to work (No 420, M2) of Khayyām) and algebraic,

M2. A Geometric Problem (Mas'ala handasiyya) - Cairo (riyāda 898/18), Leiden (14/17), New York (Columb. Smith 47). The treatise was written in Hamadan in 1209 as answer to the question of the head of the famous madrasa Niẓāmiyya at Baghdad. Research: Rashed [16], Suter [17] (33-36).

M3. Treatise on two Lines which Approach but do not Meet (Risāla fi'l-khaṭṭayn alladhayn yaqrubān wa lā yaltaqiyān) - Istanbul (SM AS 2646/2). Description of the manuscript: SHIM (490). Russian translation by Krasnova: Krasnova and Tagi-zade [1] (65-71). Proof that the product of distances from a point of an equilateral hyperbola to asymptotes is constant, that is, that equation of this hyperbola referred to asymptotes is $xy = \text{const}$.

A1. Book on the Knowledge of the Plane Astrolabe and its Operations (Kitāb fi ma'rifat al-aṣṭurlāb al-musaṭṭah wa'l-'amal bihī) - Istanbul (TK 3505/2), Leiden (591/1).

A2. Treatise on the Linear Astrolabe (Risāla fi'l-aṣṭurlāb al-khaṭṭī) - Istanbul (TK 3342/7, 3464/9), London (5479/3). Research: Carra de Vaux [5] (according to the exposition of al-Marrākushī in (No 592, A1), Michel [1] (115-122), [2], Tagi-zade and Vahabov [1] (198-202). Treatise in 2 parts: 1) construction of the astrolabe, 2) operations with it. Linear astrolabe, "stick of al-Ṭūsī" (ʿaṣā al-Ṭūsī) is a graduate ruler on which some points of a meridian and other diameters of the regular astrolabe are marked. Three threads are fastened to this ruler and one of them has a load at its end.

Mt1. Treatise on Celestial Traces (Risāla fi'l-āthār al-'ulwiyya) - is quoted in (No 674, Ph1) by al-Fārisī [1] (II 270).

SHARAF AL-DĪN AL-ṬŪSĪ ET LE POLYGONE DE NEWTON

CHRISTIAN HOUZEL

Dès le début, l'algèbre a été conçue par son inventeur al-Khwārizmī comme une science destinée à résoudre des problèmes aussi bien numériques que géométriques. Ainsi l'algèbre s'est-elle trouvée tout naturellement à la charnière de relations qui la faisaient s'articuler d'un côté avec l'arithmétique et de l'autre avec la géométrie. L'articulation avec l'arithmétique touche aussi bien l'arithmétique comme technique de calcul et système d'algorithmes que la théorie des nombres; elle a été étudiée en profondeur dans l'ouvrage, maintenant classique, de Roshdi Rashed *Entre arithmétique et algèbre*.¹ L'articulation avec la géométrie s'est développée dans une autre école des mathématiques arabes, à laquelle nous voudrions consacrer cet article; sa réalisation la plus remarquable est due à Sharaf al-Dīn al-Ṭūsī à la fin du XII^e siècle.²

Le lien entre l'algèbre et la géométrie apparaît sous une double forme: d'une part on exprime certains problèmes géométriques sous forme d'équations algébriques, étendant le programme d'al-Khwārizmī à des degrés plus grands que 2; d'autre part on utilise des constructions géométriques pour résoudre les équations algébriques. Pour la résolution des équations du troisième degré, ces constructions sont des intersections de deux coniques.

Les problèmes géométriques que nous allons considérer sont bien ancrés dans la tradition grecque; ce sont ceux que Pappus qualifie de *solides*.³ Ils étaient résolus au moyen de νεύσεις ou *inclinaisons* ou bien par intersection de deux coniques; Pappus

¹ *Entre arithmétique et algèbre: Recherches sur l'histoire des mathématiques arabes* (Paris, 1984).

² *Sharaf al-Dīn al-Ṭūsī, Œuvres mathématiques. Algèbre et géométrie au XII^e siècle*, éd. R. Rashed, Collection Sciences et Philosophie Arabes. Textes et Études, 2 vol. (Paris, 1986).

³ *Pappi Alexandrini Collectionis quae supersunt etc.*, éd. F. Hultsch, 3 vol. (Berlin, 1876-1878); trad. française P. Ver Eecke, 2 vol. (Paris, 1933).

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المطبوع ، الجزء الثالث ، ص. 527 ، 1993 القاهرة .

Amr + Muzaffer

* الطوسي (المظفر بن محمد ، شرف الدين) كان حيا سنة 576هـ / 1180م .

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